Analytical Procedures for Determining Stiffness of CLT Elements in Bending

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ABSTRACT

Various methods have been developed or adopted for the determination of design properties of CLT panels. Some of these methods are experimental in nature while others are analytical. Some methods even involve a combination of both empirical and analytical approaches based on model testing. Experimental evaluation involves determination of flexural properties by testing full-size panels or sections of panels with a specific span-to-depth ratio. The problem with the experimental approach is that every time the lay-up, type of material, or any of the manufacturing parameters change, more testing is needed to evaluate the bending properties of such products. An analytical approach, once verified with the test data, offers a more general and less costly alternative. Such an analytical approach can generally predict the strength and stiffness properties of CLT panels based on the material properties of the laminate planks that make up the CLT panel. This paper briefly summarizes the two most common analytical approaches accepted by CLT manufacturers and designers and summarizes their strengths and limitations. The first one is based on the Mechanically Jointed Beams Theory (also called the Gamma Method) and is available in Annex B of the Eurocode 5. The second one is the Shear Analogy Method that has been developed in Europe to be applicable for solid panels with cross layers. Unlike the Gamma Method, the Shear Analogy Method takes into account the shear deformation of the longitudinal and the cross layers and is the most accurate and adequate for prediction of stiffness properties of the CLT panels. This method was adopted for calculating the stiffness properties of CLT panels in the ANSI/APA PRG 320 *Standard for Performance Rated Cross Laminated Timber* [1] .

INTRODUCTION

Various methods have been developed or adopted for the determination of design properties of CLT panels. Some of these methods are experimental in nature while others are analytical. For floor elements, experimental evaluation involves determination of flexural properties by testing full-size panels or sections of panels with a specific span-to-depth ratio. The problem with the experimental approach is that every time the lay-up, type of material, or any of the manufacturing parameters change, more testing is needed to evaluate the bending properties of such new products. Obviously, the analytical approach, once verified against the test data, offers a more general and less costly alternative. An analytical approach generally predicts strength and stiffness properties of CLT based on the material properties of the laminate boards that make up the CLT panel.

The most common analytical approach that has been adopted for CLT in Europe is based on the “Mechanically Jointed Beams Theory” (also named Gamma Method) and is available in Annex B of Eurocode 5 [2]. According to this theory, the “Effective Stiffness” concept is introduced and a “Connection Efficiency Factor” (γ) is used to account for the shear deformation of the perpendicular layer, with γ=1 representing completely glued member, and γ=0 for no connection at all. This approach provides a closed (exact) solution for the differential equation only for simply supported beams/panels with a sinusoidal load distribution; however, the differences between the exact solution and those for uniformly distributed load or point loads are minimal and are acceptable for engineering practice [3].

More recently, a new method called “Shear Analogy” [4,5] has been developed that can be used for solid panels with cross layers. The methodology takes into account the shear deformation of the cross layer and is not limited to a restricted number of layers within a panel. This method seems to be the most accurate and adequate for CLT panels and as such was adopted for determining the CLT panel properties in the newly developed ANSI/APA PRG 320 Standard [1]. Since the standard gives properties for panels with certain lamina thicknesses and layouts, the basics of the method presented here can be used for predicting the bending and shear stiffness of CLT panels with various configurations.

Most of the methods have focused primarily on predicting the stiffness and not the strength properties of CLT panels in flexure, since the structural design is almost always governed by serviceability criteria such as deformation and vibration. Input parameters for the analytical procedures are the material and mechanical properties of the lumber, which should be consistent with values published in material design standards.

MECHANICAL PROPERTIES OF CLT ELEMENTS USED IN FLOOR AND WALL SYSTEMS

Usually, thickness of the individual boards currently produced varies from 16 mm to 38 mm and the width varies from 63 mm to 235 mm. Boards are finger-jointed using structural adhesive for longer spans. Boards are visually or machine stress-rated and are usually kiln dried to achieve an average moisture content of 12% ± 2%. The basic mechanical properties of the boards used in CLT elements vary from one producer to another. The larger European producers use boards stress graded C24 according to EN Standards (EN 338 and EN 1912) or S10 according to DIN Standard. The equivalent in Canada would be MSR 1650Fb-1.5E lumber that gives a modulus of elasticity of about 10,300 MPa [6,7]. Some producers use lower grades for boards located in the inner layers and for transverse layers (e.g. C16 similar to No. 3 NLGA grade and C18 similar to 2&Btr NLGA grade). Wall elements may also be manufactured using lower grades of boards.

Rolling shear strength and stiffness in CLT has been identified as a key issue that may control the design and performance of CLT floor or wall systems. Since layers are stacked crosswise during the manufacturing of CLT panels, the load bearing behaviour of this planar element is affected by the material itself and by its constructive anisotropy [8]. Work performed at the University of British Columbia [9] on CLT panels built with Canadian lodgepole pine laminates has confirmed this finding. The magnitude of the effective bending stiffness of the panel and consequently the stress distribution in the layers depend largely on the rolling shear modulus of the cross-wise layers [10]. Little information, however, is available on the rolling shear properties of CLT panels or on the determination of such properties.

The rolling shear modulus will depend on many factors such as species, cross-layer density, laminate thickness, moisture content, sawing pattern configuration, size and geometry of the board's cross-section, etc. Dynamic and numerical methods have been developed recently to measure the rolling shear modulus [11]. However, there is no general agreement among researchers, manufacturers and code officials on what method should be adopted to determine rolling shear modulus and strength. There is a lack of generalised calculation or test methods that can be adopted for the determination of rolling shear properties of CLT applicable to a wide spectrum of product lay-up details. Test methods adapted from standardized shear tests for panel type products have not been found to be satisfactory since they were developed for panels with thin layers. There is a need to develop a test method and a calculation procedure to determine the rolling shear strength and modulus of CLT.

The rolling shear modulus G_R is assumed to be 1/10 of the shear modulus parallel to the grain of the boards, G_0 (i.e. $G_R \approx G_0/10$) [8]. In Europe, the rolling shear modulus G_R of CLT panels varies usually between 50 MPa and 60 MPa.

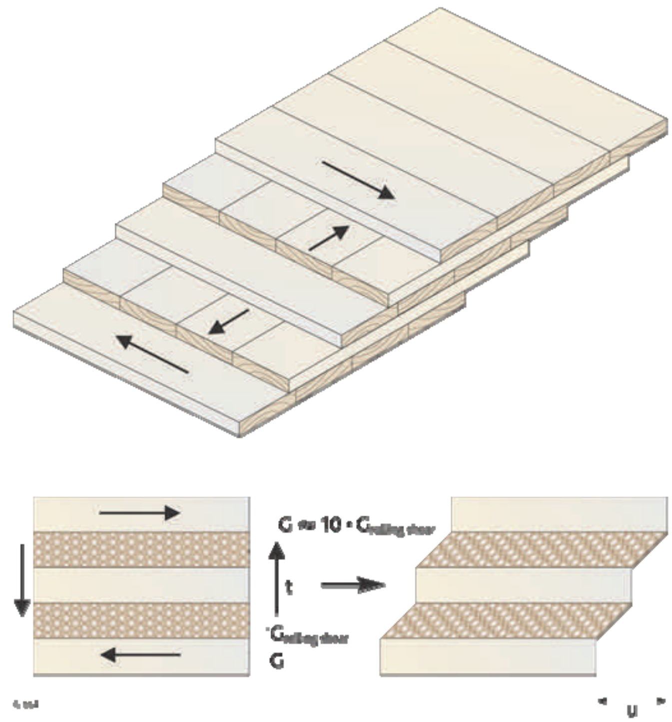

Figure 1. Rolling shear deformation of a 5-layer CLT panel

Based on experience and on the literature, the shear modulus G of wood products is generally assumed to be established between 1/12 and 1/20 of the true modulus of elasticity, i.e. $E_{true}/G_0 \approx$ 12 to 20. For example, for softwood lumber, this ratio may be assumed to be 16. Using this ratio for boards made of visually graded No. 1/No. 2 SPF sawn lumber with an MOE of 9500 MPa results in a G_0 of about 595 MPa and a rolling shear modulus of 59.5 MPa. In this case, the given magnitude of the rolling shear modulus in the literature seems to be on the conservative side. Thus, assuming a rolling shear modulus of 50 MPa in all cases, e.g. SPF, D Fir-L and Hem-Fir lumber, and MSR and visually graded boards, is on the conservative side. Please see Figure 1 for an illustration of the rolling shear deformation behaviour of a 5-layer CLT cross-section.

It is suggested that the shear deformation of CLT panels loaded uniformly may be neglected for elements having a span-to-depth ratio (*l/d*) higher than 20 [8]. Other literature as well as CLT panel producers give as a boundary condition a minimum span-to-depth ratio of 30 before neglecting the shear deformation of the panel. This is also the ratio that is suggested for use in Canada until further research in this area is conducted. However, one should always be careful about setting these boundaries. Lower span-to-depth ratios tend to be uneconomical and have higher influence of shear deformations, while in panels with larger ratios the structural performance may be controlled by the vibration properties and probably creep de-

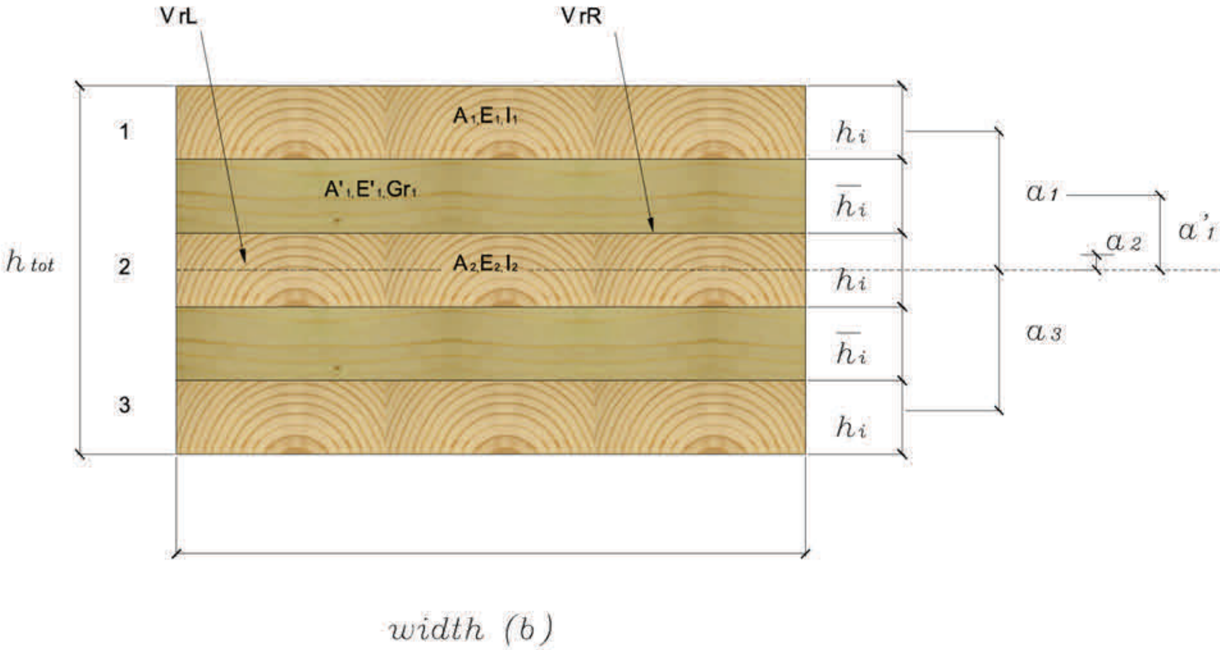

Figure 2. Cross-section of CLT panel with five layers

formation. According to preliminary calculations by the authors of this paper using the Shear Analogy Method, for a slab with a span-to-depth ratio of 30, the contribution of shear deformation is about 11% while it is 22% for a slab with a ratio of 20.

ANALYTICAL METHODS FOR CLT ELEMENTS USED IN FLOOR SYSTEMS

This section provides more detailed information about two of the most commonly used design methods: the Mechanically Jointed Beams Theory (Gamma Method), and the Shear Analogy Method.

Mechanically Jointed Beams Theory (Gamma Method)

This method was originally developed for beams (e.g., I or T beams) connected with mechanical fasteners with stiffness *K* uniformly spaced at distance *s* along the length of the beams. This method, also named the Gamma Method (γ-method) was developed by Professor Karl Möhler [12]. According to this method, the stiffness properties of the mechanically jointed beams are defined using the Effective Bending Stiffness (EI_{eff}) that depends on the section properties of the beams and the connection efficiency factor γ. The γ.factor depends on the slip characteristics of the fasteners (*s/K* ratio), being zero for no mechanical connection between the beams and equalling unity for rigidly connected (glued) beams.

Since CLT panels are glued products with no mechanical joints present, some modifications were needed for the theory to be applicable to CLT panels. If we assume that only boards oriented in the longitudinal direction are carrying the load in bending, then we can account for the rolling shear stiffness (or deformability) of the cross layers as stiffness (or deformation) caused by "imaginary fasteners" connecting the longitudinal layers. In other words, the longitudinal layers of the CLT panels are taken as "beams" connected with "mechanical fasteners" that have stiffness equal to that of the rolling shear deformation of the cross layers (Figure 1). In this case, the s/K_i ratio for "fasteners" at each interface "*i*" in the equation for determining the factor should be replaced with the rolling shear slip (shear deformation between load carrying layers) according to Equation 1.

$$\frac{s}{K_i} = \frac{\bar{h}_i}{G_R \cdot b} \tag{1}$$

where:

G_R = shear modulus perpendicular to the grain (rolling shear modulus)

$\bar{h}_i$ = thickness of board layers in direction perpendicular to the action

b = width of the panel (normally 1 meter)

s = spacing between mechanical fasteners (but not present in glued CLT)

K_i = slip modulus of mechanical fasteners (but not present in glued CLT)

The mechanically jointed beams theory is derived using simple bending theory; therefore, all its basic assumptions are valid. Shear deformations are neglected in the "beams" (i.e., longitudinal layers of the CLT slab) and are included only for the cross layers by evaluating the rolling shear deformation. This approach provides a closed solution for the differential equation only for simply supported beams/panels with a sinusoidal (or uniform) load distribution giving a moment *M = M(x)* varying in a sinusoidal or parabolic function. However, the solutions for uniformly distributed loads or for point loads differ from the exact solution by less than 3% and are deemed appropriate for engineering practice [3].

The mechanically jointed beams theory assumes that CLT elements are simply supported and have a span of "*l*". For cantilever CLT slabs, it is suggested that the length *l* to be used in the calculations should be equal to two times the cantilever length l_c. To determine the Effective Bending Stiffness (EI_{eff}) in continuous multi-supported beams, two approaches are suggested: a simplified procedure, and an iterative procedure. Since the γ factor (and therefore the effective stiffness) value depends on the length of the beam between the two zero-moment points (inflection points), according to the simplified procedure one can take the span in calculations to be equal to 0.8*l*. In the iterative procedure, one can start by considering the EI_{eff} along the length of the beam calculated using a certain length *l* (say 0.8*l*) and use a simple computer program to determine the points of inflection for a beam with that EI_{eff}. Then, by obtaining the new length between deflection points, one should re -calculate the EI_{eff} and do the analysis again. Usually after only a few iterations a stable solution for the EI_{eff} can be obtained.

As previously mentioned, rolling shear modulus G_R can be assumed to be 1/10 of the shear modulus parallel to the grain of the boards, G_0 (i.e. $G_R \approx G_0/10$). Most common values of G_R for spruce vary from 40 to 80 MPa.

Bending Strength and Stiffness for Loads Perpendicular to the Plane

The evaluation process of CLT panels in most ETA product approvals in Europe employs a hybrid approach by using a mix of analytical models and mechanical testing. Bending strength of the slab needs to be defined in relation to the effective section modulus S_{eff} of the CLT element. The bending strength should then be calculated from the bending test results using the effective section modulus. The expression for the effective section modulus is shown in Equation 2.

$$S_{eff} = \frac{2 \cdot I_{eff}}{h_{tot}} = \frac{I_{eff}}{0.5 x h_{tot}} \qquad (2)$$

where:

S_{eff} = effective section modulus

I_{eff} = effective moment of inertia (Figure 2)

h_{tot} = total depth of the panel

The effective bending stiffness can be obtained using the Equation 3.

$$EI_{eff} = \sum_{i=1}^{n} \left(E_i \cdot I_i + \gamma_i \cdot E_i \cdot A_i \cdot a_i^2 \right) \qquad (3)$$

Where $0<\gamma\leq1$ (γ=1 for rigid connection and γ=0 for no connection. But typically γ may vary from 0.85 to 0.99).

Shear Analogy Method (by Kreuzinger)

This calculation method developed by Kreuzinger [5] is the most precise design method for CLT [13]. This method is used, with the help of a plane frame analysis program, to consider the different moduli of elasticity and shear moduli of single layers for nearly any system configuration (e.g., number of layers, span-to-depth ratio). The effect of shear deformations is not neglected. In the shear analogy method, the characteristics of a multi-layer cross-section or surface (such as multi-layer CLT panels) are separated into two virtual beams A and B. Beam A is given the sum of the inherent flexural strength of the individual plies along their own neutral axes, while beam B is given the "Steiner" points part of the flexural strength, the flexible shear strength of the panel, as well as the flexibility of all connections. These two beams are coupled with infinitely rigid web members, so that an equal deflection between beams A and B is obtained. By overlaying the bending moment and shear forces (stresses) of both beams, the end result for the entire cross-section can be obtained (Figure 3).

Beam A is assigned a bending stiffness equal to the sum of the inherent bending stiffness of all the individual layers or individual cross-sections as shown in Equation 4.

$$B_A = \sum_{i=1}^{n} E_i \cdot I_i = \sum_{i=1}^{n} E_i \cdot b_i \cdot \frac{h_i^3}{12} \qquad (4)$$

where:

$B_A = (EI)_A$

b_i = width of each individual layer, usually taken as 1 m for CLT panels

h_i = thickness of each individual layer

The bending stiffness of beam B is calculated using Steiner's theorem (given as the sum of the Steiner points of all individual layers).

$$B_B = \sum_{i=1}^{n} E_i \cdot A_i \cdot z_i^2 \qquad (5)$$

where:

$B_B = (EI)_B$

z_i = the distance between the centre point of each layer and the neutral axis

Additionally, beam *B* contains the shear stiffness and the stiffness of the flexible connections, if they exist. The shear stiffness of beam *B*, S_B, is $(GA)_B$ and can be calculated as:

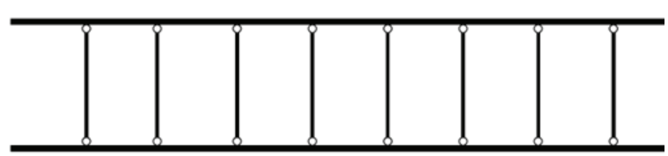

Figure 3. Beam differentiation using the shear analogy method: Beam A at the top with bending stiffness $(EI)_A = B_A$, and shear stiffness $(GA)_A = S_A \sim \infty$; Web members in the middle with infinite axial rigidity; and Beam B at the bottom with bending stiffness $(EI)_B = B_B$, and shear stiffness $(GA)_B = S_B$. [5]

$$\frac{1}{S_B}=\frac{1}{a^2}\cdot\left[\sum_{i=1}^{n-1}\frac{1}{k_i}+\frac{h_1}{2\cdot G_1\cdot b_1}+\sum_{i=1}^{n-1}\frac{h_i}{G_i\cdot b_i}+\frac{h_n}{2\cdot G_n\cdot b_n}\right] \quad (6)$$

where:

$$k_i = \frac{K_i}{s_i} \quad (7)$$

is the slip of the "fasteners" between the beams.

In the above equations, the values for E_0 shall be used for the longitudinal layers while $E_{90} = E_0/30$ is suggested to be used for cross layers. Also, in the same equations, the shear modulus for the longitudinal layer should be assumed to be G, while that for the cross layers shall be, for the rolling shear, G_R. The auxiliary members have infinite flexural strength and shear strength and serve only to connect the two beams. The continuity of deflections between beams A and B ($\Delta_A = \Delta_B$) must be valid at every point.

Using the shear analogy method, the maximum deflection u_{max} in the middle of the CLT slab under a uniformly distributed load can be calculated as a sum of the contribution due to bending and to shear:

$$u_{\max} = \frac{5}{384}\cdot\frac{qL^4}{(EI)_{eff}}+\frac{1}{8}\cdot\frac{qL^2k}{(GA)_{eff}} \quad (8)$$

or in other terms:

$$u_{\max} = \frac{5}{384}\cdot\frac{qL^4}{(EI)_{eff}}\cdot\left(1+\frac{48(EI)_{eff}\,k}{5(GA)_{eff}\,L^2}\right) \quad (9)$$

which can be expressed as:

$$u_{\max} = \frac{5}{384}\cdot\frac{qL^4}{(EI)_{eff}}(\alpha+\beta) \quad (10)$$

where α = 1.0 and β can be expressed according to Equation 11, where κ (kappa) is the shear coefficient form factor equal to 1.2 (i.e. 6/5 = 1.2) [14].

$$\beta = \frac{48(EI)_{eff}\,\kappa}{5(GA)_{eff}\,L^2} \quad (11)$$

The effective bending stiffness can be obtained using Equation 12.

$$(EI)_{eff} = B_A + B_B = \sum_{i=1}^{n}E_i\cdot b_i\cdot\frac{h_i^3}{12}+\sum_{i=1}^{n}E_i\cdot A_i\cdot z_i^2 \quad (12)$$

The effective shear stiffness can be obtained using Equation 13.

$$(GA)_{eff} = \frac{a^2}{\left[\left(\frac{h_1}{2\cdot G_1\cdot b}\right)+\left(\sum_{i=2}^{n-1}\frac{h_i}{G_i\cdot b_i}\right)+\left(\frac{h_n}{2\cdot G_n\cdot b}\right)\right]} \quad (13)$$

In the case of a concentrated force P in the middle of the span of the CLT slab, the equation for the maximum deflection is given as:

$$\begin{aligned} u_{\max} &= \frac{1}{48}\cdot\frac{PL^3}{(EI)_{eff}}+\frac{1}{4}\cdot\frac{PL}{(GA)_{eff}/\kappa} \\ &= \frac{1}{48}\cdot\frac{PL^3}{(EI)_{eff}}\left(1+\frac{12\cdot(EI)_{eff}\,\kappa}{(GA)_{eff}\,L^2}\right) \end{aligned} \quad (14)$$

which can be expressed as:

$$u_{\max} = \frac{1}{48}\cdot\frac{PL^3}{(EI)_{eff}}(\alpha+\beta) \quad (15)$$

where α = 1.0 and β can be expressed according to the Equation 16, where k (kappa) is the shear coefficient form factor equal to 1.2 (6/5 = 1.2).

$$\beta = \frac{12(EI)_{eff}\,\kappa}{(GA)_{eff}\,L^2} \quad (16)$$

Simplified Design Methods for Calculating Out-of-Plane Bending and Shear Strength

The next equations are simplified design methods proposed for calculating the capacity in bending and in shear of a CLT element acting as a floor or roof.

Bending Strength

The bending stress σ may be expressed as:

$$\sigma = M\cdot y\cdot\frac{(E_1)}{(EI)_{eff}} \quad (17)$$

The maximum stress will occur for $y=h_{tot}/2$, so Equation 17 can be expressed as:

$$\sigma_{\max} = M\cdot 0.5h_{tot}\cdot\frac{(E_1)}{(EI)_{eff}} \quad (18)$$

If CSA O86 [7] design analogy is used:

$$\sigma_{\max} \le \phi\cdot F_b \quad (19)$$

and determine the factored moment bending resistance M_r in terms of the specified bending strength F_b as:

$$M_r = \phi\cdot F_b\cdot\frac{(EI)_{eff}}{E_1}\cdot\frac{1}{0.5h_{tot}} \quad (20)$$

where E_1 is the modulus of elasticity of the outer longitudinal layer in tension and $(EI)_{eff}$ is determined according to the previous section.

When the modulus of elasticity of all longitudinal layers is equal, then Equation 20 can be expressed as:

$$M_r = \phi\cdot F_b\cdot\frac{I_{eff}}{0.5h_{tot}} \quad (21)$$

Shear Strength

Shear strength of CLT panels can be calculated using:

$$\tau_v = \frac{1.5 \cdot V}{c \cdot A_{gross}} \tag{22}$$

where coefficient *c* is a reduction factor calculated as:

$$c = \frac{I_{eff}}{I_{gross}} \tag{23}$$

If the CSA O86 design analogy was used:

$$V_r = \phi \cdot F_v \cdot \frac{2}{3} \cdot A \cdot \frac{I_{eff}}{I_{gross}} \tag{24}$$

EXAMPLES OF CLT PROPERTIES USING THE SHEAR ANALOGY METHOD (KREUZINGER)

In this section, examples for calculating the bending and shear stiffness of a typical 5-layer CLT panel in bending are given.

True Bending Stiffness (EI_{eff}) of a Five-Layer CLT Panel

A cross section of a typical 5-layer CLT panel with a total thickness of 140 mm is given in Figure 4. Important dimension and properties for calculating the stiffness values as shown in Figure 4 are:

h_1 = 32 mm, E_0 = 11000 MPa, E_{90} = 370 MPa (≈ 11000/30)

h_2 = 21 mm, E_0 = 7000 MPa, E_{90} = 230 MPa (≈ 7000/30)

h_3 = 34 mm, E_0 = 7000 MPa, E_{90} = 230 MPa (≈ 7000/30)

h_4 = 21 mm, E_0 = 7000 MPa, E_{90} = 230 MPa (≈ 7000/30)

h_5 = 32 mm, E_0 = 11000 MPa, E_{90} = 370 MPa (≈ 11000/30)

$h_{total} = h_1 + h_2 + h_3 + h_4 + h_5$ = 140 mm and b = 1000 mm

As shown in the Shear Analogy Method, the true bending stiffness (EI_{eff}) of the CLT panel can be calculated according to the formula:

$$(EI)_{eff} = B_A + B_B = \sum_{i=1}^{n} E_i \cdot b_i \cdot \frac{h_i^3}{12} + \sum_{i=1}^{n} E_i \cdot A_i \cdot z_i^2$$

<u>Step 1.</u> Determine location of the Neutral Axis, *Z*

(Note that for symmetric panels and same *E*, $Z = h_{tot}/2$)

$$E_1 A_1 = E_1 \cdot b \cdot h_1 = 11000 \cdot 1000 \cdot 32 = 3.520 x10^8\, N$$
$$E_2 A_2 = E_2 \cdot b \cdot h_2 = 230 \cdot 1000 \cdot 21 = 4.830 x10^6\, N$$
$$E_3 A_3 = E_3 \cdot b \cdot h_3 = 7000 \cdot 1000 \cdot 34 = 2.380 x10^8\, N$$
$$E_4 A_4 = E_4 \cdot b \cdot h_4 = 230 \cdot 1000 \cdot 21 = 4.830 x10^6\, N$$
$$E_5 A_5 = E_5 \cdot b \cdot h_5 = 11000 \cdot 1000 \cdot 32 = 3.520 x10^8\, N$$

$$\sum_{i=1}^{5}(E_i A_i) = 3.520x10^8 + 4.830x10^6 + 2.380x10^8 + 4.830\, x10^6 + 3.520\, x10^8$$

$$\sum_{i=1}^{5}(E_i A_i) = 9.517\, x10^8\, N$$

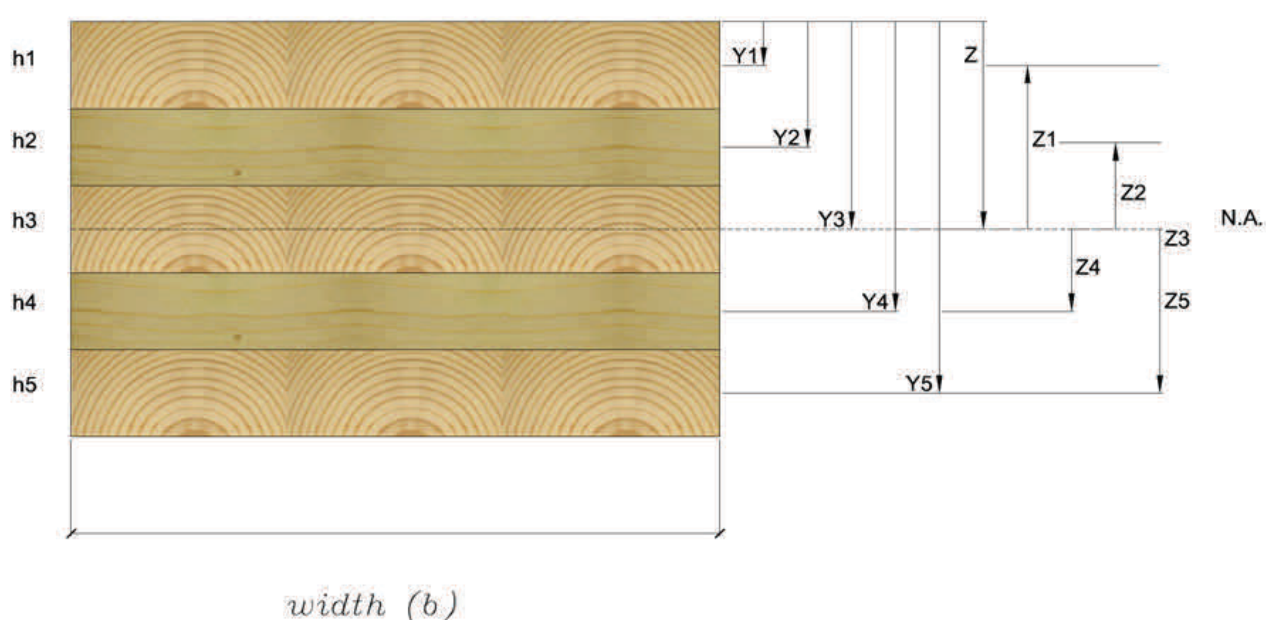

Figure 4. Cross section designations in a five-layer CLT panel

Then:

$$Y_1 = \frac{h_1}{2} = \frac{32}{2} = 16mm$$

$$Y_2 = h_1 + \frac{h_2}{2} = 32 + \frac{21}{2} = 42.5mm$$

$$Y_3 = h_1 + h_2 + \frac{h_3}{2} = 32 + 21 + \frac{34}{2} = 70mm$$

$$Y_4 = h_1 + h_2 + h_3 + \frac{h_4}{2} = 32 + 21 + 34 + \frac{21}{2} = 97.5mm$$

$$Y_4 = h_1 + h_2 + h_3 + h_4 + \frac{h_5}{2} = 32 + 21 + 34 + 21 + \frac{32}{2} = 124mm$$

$$E_1 A_1 y_1 = 3.520\,x10^8 \cdot 16 = 5.632\,x10^9\, N \cdot mm$$

$$E_2 A_2 y_2 = 4.830\,x10^6 \cdot 42.5 = 2.053\,x10^8\, N \cdot mm$$

$$E_3 A_3 y_3 = 2.380\,x10^8 \cdot 70 = 1.666\,x10^{10}\, N \cdot mm$$

$$E_4 A_4 y_4 = 4.830\,x10^6 \cdot 97.5 = 4.709\,x10^8\, N \cdot mm$$

$$E_5 A_5 y_5 = 3.520\,x10^8 \cdot 124 = 4.365\,x10^{10}\, N \cdot mm$$

$$\sum_{i=1}^{5}(E_i A_i) \cdot Y_i = 5.632x10^9 + 2.053x10^8 + 1.666x10^{10} + 4.709x10^8 + 4.365x10^{10}$$

$$\sum_{i=1}^{5}(E_i A_i) \cdot Y_i = 6.662\,x10^{10}\, N \cdot mm$$

$$Z = \frac{\sum_{i=1}^{5}(E_i A_i) \cdot Y_i}{\sum_{i=1}^{5}(E_i A_i)} = \frac{6.662x10^{10}}{9.517x10^8} = 70mm$$ and,

$$Z_1 = Z - \frac{h_1}{2} = 70 - \frac{32}{2} = 54mm$$

$$Z_2 = Z - h_1 - \frac{h_2}{2} = 70 - 32 - \frac{21}{2} = 27.5mm$$

$$Z_3 = Z - h_1 - h_2 - \frac{h_3}{2} = 70 - 32 - 21 - \frac{34}{2} = 0mm$$

$$Z_4 = -Z_2 = -27.5mm$$

$$Z_5 = -Z_1 = -54mm$$

Step 2. Calculation of $B_A = \sum_{i=1}^{n} E_i \cdot I_i = \sum_{i=1}^{n} E_i \cdot b_i \cdot \frac{h_i^3}{12}$

$$E_1 I_{a,1} = E_1 \frac{b \cdot h_1^3}{12} = 11000 \cdot \frac{1000 \cdot 32^3}{12} = 3.004x10^{10}\, N \cdot mm^2$$

$$E_2 I_{a,2} = E_2 \frac{b \cdot h_2^3}{12} = 230 \cdot \frac{1000 \cdot 21^3}{12} = 1.775x10^8\, N \cdot mm^2$$

$$E_3 I_{a,3} = E_3 \frac{b \cdot h_3^3}{12} = 7000 \cdot \frac{1000 \cdot 34^3}{12} = 2.293x10^{10}\, N \cdot mm^2$$

$$E_4 I_{a,4} = E_4 \frac{b \cdot h_4^3}{12} = 230 \cdot \frac{1000 \cdot 21^3}{12} = 1.775x10^8\, N \cdot mm^2$$

$$E_5 I_{a,5} = E_5 \frac{b \cdot h_5^3}{12} = 11000 \cdot \frac{1000 \cdot 32^3}{12} = 3.004x10^{10}\, N \cdot mm^2$$

Then,

$$B_A = 3.004x10^{10} + 1.775x10^8 + 2.293x10^{10} + 1.775x10^8 + 3.004x10^{10}$$

$$B_A = 8.337x10^{10}\, N \cdot mm^2$$

Step 3. Calculation of $B_B = \sum_{i=1}^{n} E_i \cdot A_i \cdot z_i^2$

$$E_1 I_{b,1} = E_1 A_1 \cdot Z_1^2 = 11000 \cdot 1000 \cdot 32 \cdot 54^2 = 1.026x10^{12}\, mm^4$$

$$E_2 I_{b,2} = E_2 A_2 \cdot Z_2^2 = 230 \cdot 1000 \cdot 21 \cdot 27.5^2 = 3.653x10^9\, mm^4$$

$$E_3 I_{b,3} = E_3 A_3 \cdot Z_3^2 = 7000 \cdot 1000 \cdot 34 \cdot 0 = 0mm^4$$

$$E_4 I_{b,4} = E_4 A_4 \cdot Z_4^2 = 230 \cdot 1000 \cdot 21 \cdot (-27.5^2) = 3.653x10^9\, mm^4$$

$$E_5 I_{b,5} = E_5 A_5 \cdot Z_5^2 = 11000 \cdot 1000 \cdot 32 \cdot (-54^2) = 1.026x10^{12}\, mm^4$$

Then,

$$B_B = 1.026\,x10^{12} + 3.653\,x10^9 + 0 + 3.653\,x10^9 + 1.026\,x10^{12}$$

$$B_B = 2.059\,x10^{12}\, N \cdot mm^2$$

Finally:

$$(EI)_{eff} = B_A + B_B = 8.337x10^{10} + 2.059x10^{12}$$

$$(EI)_{eff} = 2.143x10^{12}\, N \cdot mm^2$$

Shear Stiffness (GA_{eff}) of a Five-Layer CLT Panel

We assume the same cross section (Figure 4) for this example as well with the following properties:

h_1 = 32 mm G_0 = 690 MPa G_{90} = 69 MPa (≈ $G_0/10$)

h_2 = 21 mm G_0 = 440 MPa G_{90} = 44 MPa (≈ $G_0/10$)

h_3 = 34 mm G_0 = 440 MPa G_{90} = 44 MPa (≈ $G_0/10$)

h_4 = 21 mm G_0 = 440 MPa G_{90} = 44 MPa (≈ $G_0/10$)

h_5 = 32 mm G_0 = 690 MPa G_{90} = 69 MPa (≈ $G_0/10$)

$h_{total} = h_1 + h_2 + h_3 + h_4 + h_5$ = 140 mm and b = 1000 mm

$$a = h_{total} - \frac{h_1}{2} - \frac{h_n}{2} \qquad where \quad n = 5$$

$$a = 140 - \frac{32}{2} - \frac{32}{2} = 108mm$$

$$(GA)_{eff} = \frac{a^2}{\left[\left(\frac{h_1}{2 \cdot G_1 \cdot b}\right) + \left(\sum_{i=2}^{n-1} \frac{h_i}{G_i \cdot b_i}\right) + \left(\frac{h_n}{2 \cdot G_n \cdot b}\right)\right]}$$

$$(GA)_{eff} = \frac{a^2}{\left[\left(\frac{h_1}{2 \cdot G_1 \cdot b}\right) + \left(\sum_{i=2}^{4} \frac{h_i}{G_i \cdot b_i}\right) + \left(\frac{h_5}{2 \cdot G_5 \cdot b}\right)\right]}$$

$$(GA)_{eff} = \frac{a^2}{\left[\left(\frac{h_1}{2 \cdot G_1 \cdot b}\right) + \left(\frac{h_2}{G_2 \cdot b} + \frac{h_3}{G_3 \cdot b} + \frac{h_4}{G_4 \cdot b}\right) + \left(\frac{h_5}{2 \cdot G_5 \cdot b}\right)\right]}$$

$$(GA)_{eff} = \frac{108^2}{\left[\left(\frac{32}{2 \cdot 690 \cdot 1000}\right) + \left(\frac{21}{44 \cdot 1000} + \frac{34}{440 \cdot 1000} + \frac{21}{44 \cdot 1000}\right) + \left(\frac{32}{2 \cdot 690 \cdot 1000}\right)\right]}$$

$$(GA)_{eff} = 1.082\, x 10^7\, N$$

CONCLUSIONS

Two of the most common approaches for determination of design properties of CLT panels are presented. The most common analytical approach that has been adopted for CLT in Europe is based on the mechanically jointed beams theory that is available in Annex B of Eurocode 5 [2]. This approach provides a closed solution for the differential equation only for simply supported beams/panels with a sinusoidal load distribution. However, the solutions for uniformly distributed loads or for point loads differ from the exact solution by less than 3% and are acceptable for engineering practice [3]. More recently, a new method called the “Shear Analogy Method” [5] has been developed that can be used for solid panels with cross layers. The methodology takes into account the shear deformation of the longitudinal and the cross layers and is not limited by the number of layers within a panel. This method seems to be the most accurate and adequate for CLT panels and as such it was adopted for determining the CLT panel properties in the newly developed ANSI/APA PRG 320 Standard [1]. Since the standard gives properties for panels with certain lamina thicknesses and layouts, the basics of the method presented here can be used for predicting the bending and shear stiffness of CLT panels with various configurations.

Most of the studies conducted so far have focused primarily on predicting the stiffness and not the strength properties of CLT panels in flexure. While flexural stiffness of CLT panels is usually of greater interest for designers than the strength, since the structural design is mostly governed by serviceability criteria (i.e., deflection and vibration), from a product standard development point of view, there is a need to characterize the strength properties as well, to ensure certain minimum panel strength in service. Design methods to evaluate out-of-plane and in-plane bending strength have been proposed. The procedures to calculate the design properties should be based on material properties for the lumber published in the national design standards, and should be consistent with the design philosophy in the standards. The examples in this paper were related to CSA O86, the Canadian Standard for Engineering Design in Wood [7]. Because of these important considerations, the developed analytical method will need to be comprehensively verified against test data.

REFERENCES

[1] ANSI/APA 2011. ANSI/APA PRG 320. Standard for Performance Rated Cross-Laminated Timber. American National Standards Institute. NY. Published by APA–The Engineered Wood Association, Tacoma, WA.

[2] EN 2004. European Committee for Standardization. *Eurocode 5: Design of timber structures. Part 1-1: General – Common rules and rules for buildings*. EN 1995-1-1. Brussels. 124 p.

[3] Ceccotti A. 2003. Composite structures. In *Timber Engineering*, ed. S. Thelandersson and H. J. Larsen, chapter 21. West Sussex, England: John Wiley and Sons.

[4] Kreuzinger H. 1995. Mechanically jointed beams and columns. In *Timber Engineering – STEP 1*, ed. H. J. Blass et al., B11/1-8. Almere, The Netherlands: Centrum Hout.

[5] Kreuzinger H. 1999. Platten, Scheiben und Schalen – ein Berechnungsmodell für gängige Statikprogramme. *Bauen mit Holz* 1: 34-39.

[6] NLGA 2010. National Lumber Grades Authority. Special Products Standard for Fingerjoined Structural Lumber. SPS 1. New Westminster, British Columbia, Canada.

[7] CSA 2009. Canadian Standards Association. *Engineering design in wood (limit states design)*. CSA O86-09. Rexdale, ON: CSA. 222 p.

[8] Mestek, P., H. Kreuzinger, and S. Winter. 2008. Design of cross laminated timber (CLT). Paper presented at the *10th World Conference on Timber Engineering,* June 2-5, 2008, Miyazaki, Japan.

[9] Bejtka, I., F. Lam. 2008. Cross laminated timber as innovative building material. In *Proceedings of the CSCE Annual Conference*, *Québec, QC*. Montréal, QC: Canadian Society for Civil Engineering. CD-ROM.

[10] Fellmoser, P., and H. J. Blass. 2004. Influence of rolling shear modulus on strength and stiffness of structural bonded timber elements. In *Proceedings of CIB-W18 Meeting, Edinburgh, United Kingdom*, paper 37-6-5.

[11] Steiger, R., A. Gülzow, and D. Gsell. 2008. Non-destructive evaluation of elastic material properties of cross laminated timber (CLT). In *Proceedings of the COST E53 Conference, October 29-30, 2008, Delft, The Netherlands*, p. 171-182.

[12] Möhler, K. 1955. Über das Tragverhalten von Biegeträger und Druckstäben mit zusamengestzten Querschnitten und nachgiebigen Verbindungsmitteln. Habilitation thesis, Technische Hochschule, Karlsruhe.

[13] Blass H. J., P. Fellmoser. 2004. Design of solid wood panels with cross layers. In *Proceedings of the 8th World Conference on Timber Engineering, June 14 -17, 2004, Lahti, Finland*, 2:543-548.

[14] Timoshenko, S., Woinowsky-Krieger. 1959. *Theory of plates and shells*. New York: McGraw-Hill. 580 p.

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